Topic 10.1 – Understanding Rates

Rate – a special type of ratio that compares quantities with unlike units of measure, if the comparison is to 1 unit, the rate is called a **unit rate**.

Rate = 150 miles/3 hours

Unit Rate = 50 miles/1 hour

First, write the ratio as a rate. 7 Km in 4 minutes

$$\frac{7\text{ Km}}{4\text{ min}}$$

To find the unit rate, divide the first term by the second term. "Bottom into Top"

$$4\overline{)7.00}\quad 1.75$$

-4

30

-28

20

-20

0

To understand why it works, remember that you can divide the terms of any ratio by the same number to find an equal ratio.

$$\frac{7 \div 4}{4 \div 4} = \frac{1.75}{1} \qquad \frac{1.75\text{Km}}{1\text{ min}} = \text{unit rate}$$

1.75 Km in 1 minute.

More Examples:

(1.) 60 Km in 12 hours $\quad \frac{60\text{Km}}{12\text{ h}} = \frac{5\text{ Km}}{1\text{ h}}$

$$12\overline{)60}\quad 5$$

-60

0

(2) 26 cm in 13s $\quad \frac{26\text{ cm}}{13\text{ s}} = \frac{2\text{ cm}}{1\text{ s}}$

$$13\overline{)26}\quad 2$$

-26

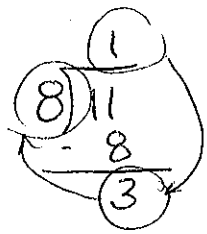
0

Topic 10.2 - Comparing Rates

1. Ethan swam 11 laps in the pool in 8 minutes. Austin swam 7 laps in the same pool in 5 minutes. Which boy swam at a faster rate? Find the unit rates to compare who swam faster.

   - Find Ethan's unit rate

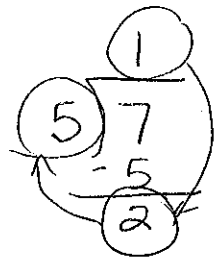
   $\frac{11 \text{ laps}}{8 \text{ mins}}$

   $1\frac{3}{8} = \frac{1.375 \text{ laps}}{1 \text{ min}}$

   - Find Austin's unit rate

   $\frac{7 \text{ laps}}{5 \text{ mins}}$

   $1\frac{2}{5} = \frac{1.4 \text{ laps}}{1 \text{ min}}$

   - Compare the unit rates

   1.4 > 1.375, so Austin swam at a faster rate.

2. Which has a faster average speed: a car that travels 600 feet in 20 seconds or a motorcycle that travels 300 feet in 12 seconds?

   $\frac{600 \text{ ft}}{20 \text{ s}}$

   $20\overline{)600}$ → 30

   - 60↓

   00

   $\frac{30 \text{ ft}}{1 \text{ s}}$

   $\frac{300 \text{ ft}}{12 \text{ s}}$

   $12\overline{)300}$ → 25

   - 24↓

   60

   - 60

   0

   $\frac{25 \text{ ft}}{1 \text{ s}}$

   ★ The car has a faster average speed! ★

3. Which unit rate is greater: 217 miles in 7 hours or 396 miles in 12 hours.

   $\frac{217 \text{ mi}}{7 \text{ h}} = 7\overline{)217}$ → 31

   - 21↓

   07

   - 7

   0

   $= \frac{31 \text{ mi}}{1 \text{ h}}$

   $\frac{396 \text{ mi}}{12 \text{ h}} = 12\overline{)396}$ → 33

   - 36↓

   36

   - 36

   0

   $= \frac{33 \text{ mi}}{1 \text{ h}}$

   This unit is greater!

Topic 10.3 – Unit Rates

How can you use a unit rate to solve a proportion?

1. A bicycle tour group travels 320 miles in 5 days. How far could they travel in 8 days if they maintained the same average speed?
   - Find a unit rate to solve problem.

   Step 1: Find the unit rate. The group traveled 320 miles in 5 days. The unit rate tells how many average miles they traveled per day.

   - Divide 320 by 5. 320/5 = 64
   - The unit rate is 64/1. The group traveled an average of 64 miles a day.

   Step 2: Use the unit rate to find how far the group could travel in 8 days.

   - 1 unit = 64 miles
   - Multiply 64 miles by 8. (HOW FAR COULD THEY TRAVEL IN ***8*** DAYS?)
   - 64 x 8 = 512
   - The group could travel 512 miles in 8 days.

2. A construction crew can spread 2 tons of gravel in 90 minutes. How long does it take them to spread 1 ton?

$$2\overline{)90} \quad 45,\ -8\downarrow,\ 10,\ -10,\ 0$$

45 minutes / 1 ton

More Examples:

① $\frac{320 \text{ mi}}{16 \text{ gal}}$

$$16\overline{)320} \quad 20,\ -32\downarrow,\ 00$$

$\frac{20 \text{ mi}}{1 \text{ gal}}$

② $\frac{2 \text{ in}}{1 \text{ yr}} = \frac{x \text{ in}}{13 \text{ yr}}$ (×13 across the numerators, ×13 across the denominators)

$\frac{26 \text{ in}}{13 \text{ yr}}$

Topic 10.4 – Unit Price

Unit Price – a unit rate that gives the price of one item.

1. Is the lunch special or the weekend special a better deal?
   - Lunch = 3 tacos for $2.40
   - Weekend = 4 tacos for $3.40

Step 1:

- Find the unit price of the lunch special.
- Write the cost of the lunch special as a rate. $2.40/3 tacos
- Divide the terms. $2.40/3 = $0.80
- The unit price for the lunch special is $0.80 per taco

Step 2:

- Find the unit price of the weekend special.
- Write the cost of the weekend special as a rate. $3.40/4 tacos
- Divide the terms. $3.40/4 = $0.85
- The unit price for the weekend special is $0.85 per taco.
- Now compare: $0.80 per taco is less than $0.85 per taco, so the lunch special is the better buy!

More Examples:

2. What is the better buy? 3 kilograms of charcoal for $7.95 or 5 kilograms of charcoal for $12.50.

$$3\overline{)7.95} = \$2.65 \qquad 5\overline{)12.50} = \$2.50$$

★ 5 Kg for $12.50 is the better buy because $2.50 is less than $2.65.

3. What is the better buy? 50 envelopes for $2.99 or 90 envelopes for $5.50.

$$50\overline{)2.9900} = 0.0598 \approx \$0.060 \qquad 90\overline{)5.500} = 0.061 \approx \$0.061$$

★ 50 envelopes for $2.99 is the better buy!

Topic 10.5 Constant Speed

Formula – a rule that uses symbols to relate quantities.

Constant Speed – when speed stays the same over time.

How are distance, rate, and time related?

Step 1:

- Use the formula.
- d = r x t
- distance = rate x time
- Substitute values for the two variables you know.

Step 2:

- Use properties of equality and inverse relationships to get the unknown variable alone on one side of the equation.

Examples:

1. Finding Distance: distance = d rate = 55 mph time = $\frac{1}{2}$ hr

$d = r \times t$

$d = 55 \times \frac{1}{2}$

$d = 27.5$ miles

$$\begin{array}{r} {}^{2}55 \\ \times \quad .5 \\ \hline 27.5 \end{array}$$

2. Finding Rate: distance = 12 mi rate = r time = 2 h

$d = r \times t$

$12 = r \times 2$

$\div 2 \quad \div 2$

$6 = r$

6 mph

3. Finding Time: distance = 96 ft rate = 32 ft/s time = t

$d = r \times t$

$96 = 32 \times t$

$\div 32 \quad 32 \div$

$3 = t$

3 s

$$32\overline{)96}\quad 3,\ -96,\ 0$$

Topic 10.6 - Converting Customary Units

A table of customary units can help you convert from one unit of measurement to another

**Remember: Large to small, multiply them all. Small to large, divide and take charge!*

Customary Units of Length:

Foot (ft) 1 ft = 12 in

Yard (yd) 1 yd = 3 ft

1 yd = 36 in

Mile (mi) 1 mi = 5,280 ft

1 mi = 1,760 yd

Customary Units of Weight:

Pound (lb) 1 lb = 16 ounces (oz)

Ton (T) 1 T = 2,000 lb

Customary Units of Capacity: *(Capacity is the volume of a container measured in liquid units.)*

Cup (c) 1 c = 8 fluid ounces (fl oz)

Pint (pt) 1 pt = 2 c

Quart (qt) 1 qt = 2 pt

Gallon (gal) 1 gal = 4 qt

Examples:

(1) 3 gal = ___ qt

3 × 4 = 12 qt

(2) 2 mi = ___ ft

2 × 5280 = 10,560 ft

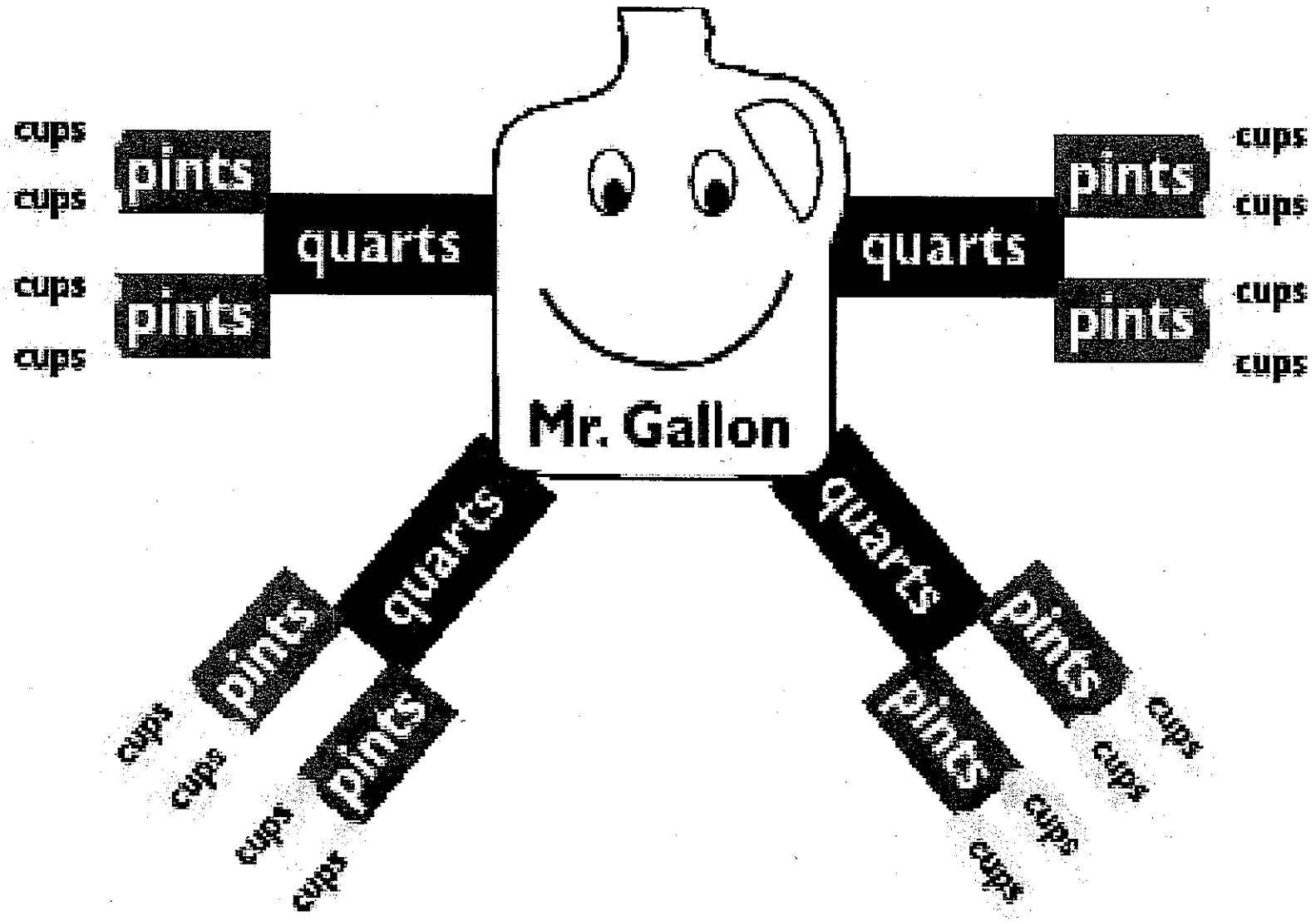
(3) 36 in = ___ ft

36 ÷ 12 = 3 ft

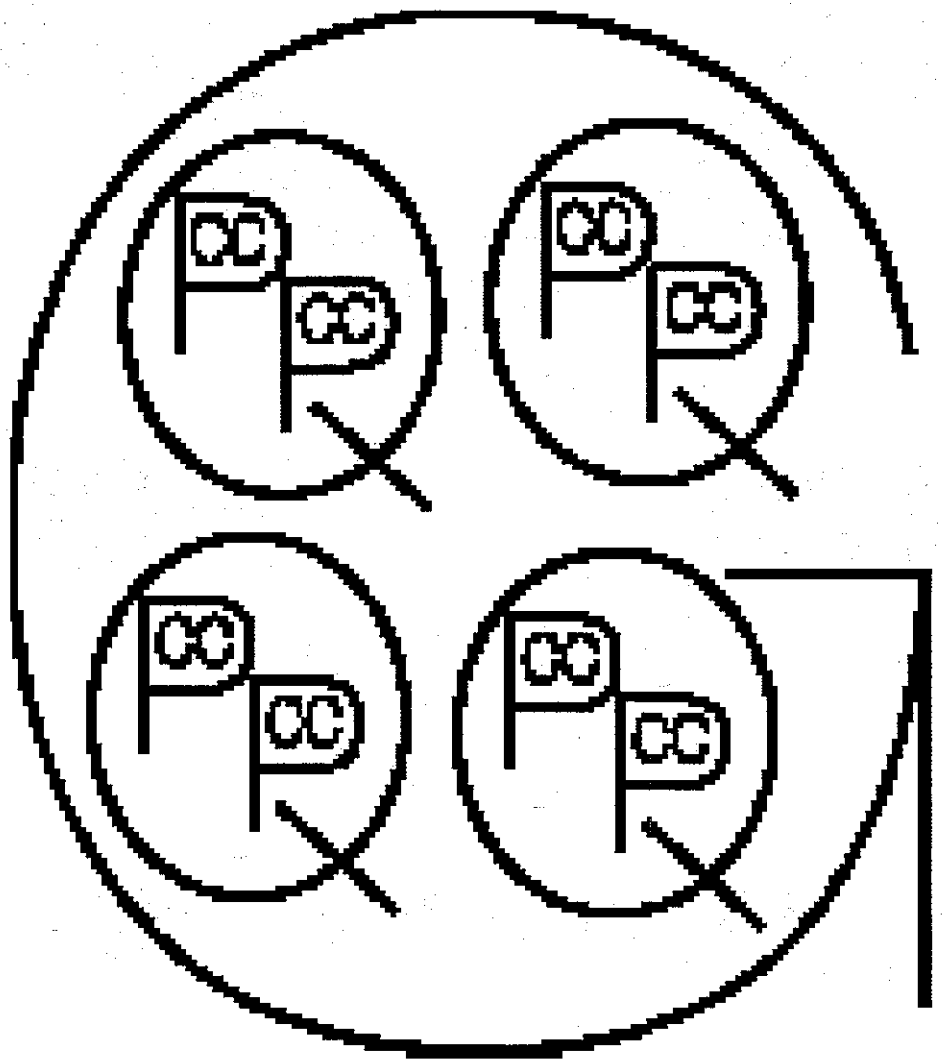
(4) 5 pt = ___ c

5 × 2 = 10 c

To help you remember...



To help you remember...



Topic 10.7 – Converting Metric Units

The Metric System is based on powers of 10

Meter (m) is the basic unit of length.

Gram (g) is the basic unit of mass.

Liter (L) is the basic unit of capacity.

The Metric System uses prefixes to describe amounts that are larger or smaller than the basic unit

Kilo – means 1,000, **Centi** – means 1/100, **Milli** – means 1/1,000

**To change from one unit to another, multiply or divide by a power of 10* OR*

★ **Move the decimal to the left or right** ★

move (R) ×10 ×10 ×10 ×10 ×10 ×10

kilo-	hecto-	deka-	Unit	deci-	centi-	milli-
km	hm	dam	**meter (m)**	dm	cm	mm
kg	hg	dag	**gram (g)**	dg	cg	mg
kL	hL	daL	**liter (L)**	dL	cL	mL

move (L) ÷10 ÷10 ÷10 ÷10 ÷10 ÷10

Examples:

① 4m = ____ cm

4.00 = 400 cm

② 300 g = ____ Kg

300. = .3 Kg

③ 2 L = ____ mL

2,000. = 2,000 mL

④ 486 cm = ____ m

486. = 4.86 m

⑤ 90.25 Kg = ____ g

90,250. = 90,250 g

⑥ 8mm = ____ cm

8. = 0.8 cm

Topic 10.8 – Problem Solving: Writing to Explain

The steepness of a ramp is the ratio of the height to the length of the base. The ramp at the right has a steepness of 3:4. What would be the length of another ramp with the same steepness if the height were 9 feet?

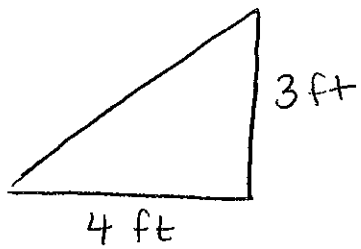
Jim's answer: 9 is 6 more than 3. If I add 6 to 4 the sum is 10. The new length is 10 feet.

Is Jim's answer correct? Explain.

Read and Understand:

Show the information you know.

Use words, pictures, numbers, and symbols to write good math explanations



Plan and Solve:

Build a bigger ramp using the smaller ramp several times.

Height: 3 + 3 + 3 = 9

Length of the base: 4 + 4 + 4 = 12

The ratio 9:12 is equal to 4:12. The correct length of the new ramp is 12 feet.

Jim's answer is not correct